Eulerian–Eulerian two-phase numerical simulation of nanofluid laminar forced convection in a microchannel

Mohammad Kalteha,b, Abbas Abbassia, Majid Saffar-Avala, Jens Harting b,c

Abstract

In this paper, laminar forced convection heat transfer of a copper–water nanofluid inside an isothermally heated microchannel is studied numerically. An Eulerian two-fluid model is considered to simulate the nanofluid flow inside the microchannel and the governing mass, momentum and energy equations for both phases are solved using the finite volume method. For the first time, the detailed study of the relative velocity and temperature of the phases are presented and it has been observed that the relative velocity and temperature between the phases is very small and negligible and the nanoparticle concentration distribution is uniform. However, the two-phase modeling results show higher heat transfer enhancement in comparison to the homogeneous single-phase model. Also, the heat transfer enhancement increases with increase in Reynolds number and nanoparticle volume concentration as well as with decrease in the nanoparticle diameter, while the pressure drop increases only slightly.

Keywords:
- Nanofluid
- Microchannel
- Two-phase
- Laminar
- Heat transfer

1. Introduction

Emerging developments in MEMS (Micro-Electro-Mechanical-Systems) make it possible to fabricate very small scale devices. On the other hand, these small scale devices can generate a high amount of heat flux that should be taken away by a cooling system to guarantee their appropriate performance. One possible way to cool these devices is the use of so-called nanofluids. A nanofluid is a suspension of nano-sized (10–100 nm) metallic and non-metallic solid particles in a conventional cooling liquid such as water, ethylene glycol, or oil. The term nanofluid for the first time was used by Choi (1995) for such a suspension. After that, many researchers focused on studying the thermophysical and also heat and fluid flow properties of nanofluids. Most of the studies concentrated on the modeling of the effective thermal conductivity of nanofluids (e.g. Xuan et al., 2003; Koo and Kleinstreuer, 2004; Feng et al., 2007). Recently, researchers have focused on the nanofluid heat and fluid flow behavior.

There are many experimental studies for nanofluids on macro and micro-scales (e.g. Wen and Ding, 2004; Heris et al., 2006; Jung et al., 2009; Wu et al., 2009). Wen and Ding (2004) investigated the heat transfer of an Al2O3–water nanofluid in the entrance region of a 4.5 mm diameter copper tube under the constant heat flux conditions. Their measurements showed enhancement in heat transfer especially in the entrance region of the tube. They described this behavior as the particle migration effect (non-uniform nanoparticle volume concentration) that reduces the thermal boundary layer thickness. For an annular tube with a 6 mm inner diameter copper tube and a 32 mm outer diameter stainless steel tube, Heris et al. (2006) studied CuO and alumina nanoparticles in water. They compared the experimental results with homogeneous model results (single-phase correlations with nanofluid effective properties) and reported that the homogeneous modeling under-estimates the heat transfer enhancement, especially in higher volume concentrations.

Jung et al. (2009) did experiments for Al2O3–water nanofluids in rectangular microchannels. The particle diameter in their experiments was 170 nm. With only 1.8% of volume concentration they reported a 32% increase of the heat transfer coefficient in comparison to single distilled water. Also, experiments on nanofluid heat transfer in trapezoidal silicon microchannels have been performed by Wu et al. (2009). For channels with a hydraulic diameter of 194.5 μm and an Al2O3–water nanofluid, they reported an increase in the Nusselt number with increasing particle concentration, Reynolds and Prandtl numbers, while the pressure drop increased slightly when compared to pure water.

For the theoretical study of the pressure-driven nanofluid heat and fluid flow commonly homogenous (single-phase) and two-phase models are used. In homogeneous modeling it is assumed that the particles and the base fluid have the same temperature and velocity and thus, the single-phase equations along with the...
appropriate effective thermophysical properties (thermal conductivity, viscosity, specific heat and density) for the nanofluid are solved. In this method, the accuracy of the models used as effective thermophysical properties is very important. Most of the theoretical studies in this field are based on the homogeneous approach (e.g. Koo and Kleinstreuer, 2005; Li and Kleinstreuer, 2008; Santra et al., 2009). In addition to the pressure-driven nanofluid flows, it is possible to use electroosmotic transport for nanofluids especially in the micro-scale. In this case, the thickness of the electrical double layer and effective electrical conductivity of the nanofluid can affect the nanofluid heat transfer behavior (Chakraborty and Padhy, 2008; Chakraborty and Roy, 2008).

In despite of the homogeneous modeling, in the two-phase modeling, the nanoparticle and the base fluid are considered as two different phases with different velocities and temperatures. In this method, the interactions between the phases are taken into account in the governing equations. There are a few studies that used two-phase approach to study nanofluids. Behzadmehr et al. (2007) used a two-phase mixture model to study the turbulent nanofluid convection inside a circular tube. Comparing with an experimental study they reported that the two-phase results are more precise than the homogeneous modeling results. However, they considered thermal equilibrium conditions (the same temperature) for the phases. Mirmasoumi and Behzadmehr (2008a) used the same method as in Behzadmehr et al. (2007) to study the mixed convection of the nanofluid in a tube. Also, Mirmasoumi and Behzadmehr (2008b) and Akbarinia and Laur (2009) investigated the nanoparticles size effect on the mixed convective heat transfer of a nanofluid using the two-phase mixture method. In both studies an increase in heat transfer with a decrease in the nanoparticles size was reported. Bianco et al. (2009) modeled the nanofluid flow and heat transfer inside a tube. They used both single-phase and two-phase methods. For the two-phase method, they implemented Lagrangian approach to model the particle motion. They reported a maximum difference of 11% between the single and two-phase results. Kurowski et al. (2009) used three different homogeneous, Eulerian–Lagrangian and mixture methods to simulate nanofluid flow inside a minichannel. Their results showed almost the same behavior for all the methods. Fard et al. (2010) studied the nanofluid heat transfer inside a tube considering both the single and two-phase methods. For a 0.2% copper–water nanofluid, they reported that the average relative error between the experimental data and single-phase model was 16% while for the two-phase method it was 8%. On the other hand, Lotfi et al. (2010) used homogeneous, two-phase Eulerian and mixture models for nanofluid flow inside a tube. They reported that among these methods, the two-phase mixture method is more precise than the others.

According to the literature, there is a non-uniform nanoparticle volume concentration distribution in the entrance region (Wen and Ding, 2004) and the homogeneous model under-estimates the observed heat transfer enhancement in the experiments (Heris et al., 2006; Behzadmehr et al., 2007; Bianco et al., 2009; Fard et al., 2010; Lotfi et al., 2010). Thus, the two-phase modeling can be an alternative method. On the other hand, the existing studies for the two-phase method do not consider the temperature difference between the phases (Behzadmehr et al., 2007; Mirmasoumi and Behzadmehr, 2008a,b; Akbarinia and Laur, 2009) or do not present detailed results on the relative velocity and temperature between the phases and the volume concentration distribution (Bianco et al., 2009; Fard et al., 2010). The amount of the relative velocity and temperature between the phases along with the nanoparticle concentration distribution can provide an estimation of the accuracy of the assuming nanofluid as a homogeneous solution.
the other hand, all the above mentioned two-phase studies are for macro-sized circular tubes and to the best of the knowledge of the authors there is no such study for microchannels. So, this paper aims to study the nanofluid laminar forced convection in a deep rectangular (parallel plate) microchannel with isothermally heated walls, using the Eulerian–Eulerian two-phase model. To do this, mass, momentum and energy conservation equations for both phases are solved with the iterative numerical methods. The two-phase results are compared with the single-phase results from the literature and then the nanoparticle size, nanoparticle concentration and Reynolds number effects on the nanofluid heat transfer behavior are studied. Also, the relative velocity and temperature between the phases and the particle volume concentration distribution in the field are investigated. To the best knowledge of the authors, this is the first paper reporting the detailed two-phase nanofluid modeling in a microchannel that considers different velocity and temperatures for the phases.

2. Governing equations

The geometry of the present problem is shown in Fig. 1. It consists of a parallel plate microchannel with height 200 μm and the length $L$ is 100 times larger than the height ($L/H = 100$). The origin of the Cartesian coordinate system is considered to be at the plate symmetry axis and only the top half of the channel is used for numerical simulation. In this problem laminar nanofluid flow that is a mixture of water and copper nanoparticles enters the channel with a uniform velocity and temperature and exchanges heat with the isothermal microchannel walls.

Considering the laminar, steady state and two-dimensional Eulerian–Eulerian two-phase model for the nanofluid, the governing mass, momentum and energy equations for the particle and base liquid phases can be written as follows (Fluent user’s guide, 2006; Hao and Tao, 2004).

2.1. Continuity equations

$$\frac{\partial (\rho_1 u_1)}{\partial x} + \frac{\partial (\rho_1 v_1)}{\partial y} = 0$$

(1)

$$\frac{\partial (\rho_p u_p)}{\partial x} + \frac{\partial (\rho_p v_p)}{\partial y} = 0$$

(2)

where $x$, $y$, $u$, $v$, $\rho$ and $\phi$ are axial and vertical direction, axial and vertical velocity, density and volume concentration, respectively.

Also, subscripts $l$ and $p$ stand for liquid and particle phases, respectively. According to the volume concentration definition we have

$$\phi_l + \phi_p = 1$$

(3)

2.2. Momentum equations

Momentum equations in the $x$-direction are

$$\frac{\partial (\rho_1 u_1 u_1)}{\partial x} + \frac{\partial (\rho_1 v_1 u_1)}{\partial y} = -\phi_l \frac{\partial p_l}{\partial x} + \frac{\partial (\phi_l \mu_l \frac{\partial u_1}{\partial x})}{\partial x}$$

$$+ \frac{\partial (\phi_l \mu_l \frac{\partial u_1}{\partial y})}{\partial y} + (F_{d,l})_x + (F_{em})_x$$

(4)

and

$$\frac{\partial (\rho_p u_p u_p)}{\partial x} + \frac{\partial (\rho_p v_p u_p)}{\partial y} = -\phi_p \frac{\partial p_p}{\partial x} + \frac{\partial (\phi_p \mu_p \frac{\partial u_p}{\partial x})}{\partial x}$$

$$+ \frac{\partial (\phi_p \mu_p \frac{\partial u_p}{\partial y})}{\partial y} + (F_{d,p})_x + (F_{col})_x$$

(5)

Here, $p$, $\mu$, $(F_{d,l})_x$, $(F_{em})_x$, and $(F_{col})_x$ are the pressure, viscosity, drag, virtual mass (added mass) and particle–particle interaction forces in the $x$-direction, respectively. Due to the very small size of the nanoparticles, the lift force between the phases is neglected in the present study.

Momentum equations in the $y$-direction can be written as follows:

$$\frac{\partial (\rho_1 u_1 v_1)}{\partial x} + \frac{\partial (\rho_1 v_1 v_1)}{\partial y} = -\phi_l \frac{\partial p_l}{\partial y} + \frac{\partial (\phi_l \mu_l \frac{\partial v_1}{\partial x})}{\partial y}$$

$$+ \frac{\partial (\phi_l \mu_l \frac{\partial v_1}{\partial y})}{\partial y} + (F_{d,l})_y + (F_{em})_y$$

(6)

$$\frac{\partial (\rho_p u_p v_p)}{\partial x} + \frac{\partial (\rho_p v_p v_p)}{\partial y} = -\phi_p \frac{\partial p_p}{\partial y} + \frac{\partial (\phi_p \mu_p \frac{\partial v_p}{\partial x})}{\partial y}$$

$$+ \frac{\partial (\phi_p \mu_p \frac{\partial v_p}{\partial y})}{\partial y} + (F_{d,p})_y + (F_{col})_y$$

(7)

where $(F_{d,l})_y$, $(F_{em})_y$, and $(F_{col})_y$ are the drag, virtual mass (added mass) and particle–particle interaction forces in the $y$-direction, respectively. Due to the small size of the channel, the gravitational force is neglected in the present study.

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![Fig. 1. Geometry of the isothermally-heated parallel plate microchannel with length $L$ and height $H$. Nanofluid enters the channel with uniform velocity and temperature and leaves the channel while it is fully developed.](image-url)
The drag force between the phases is calculated as

$$F_d = -\beta \left( \vec{V} - \vec{V}_p \right)$$ \hspace{1cm} (8)

where $\vec{V}$ is the velocity vector.

The friction coefficient $\beta$ is calculated according to the particle volume concentration range. For very dilute two-phase flows with particle diameter $d_p$, the friction coefficient is (Syamal and Gidas, 1985)

$$\beta = \frac{3}{4} C_d \frac{\partial \phi}{\partial y} \left| \vec{V} - \vec{V}_p \phi \right|^{-2.65}$$ \hspace{1cm} (9)

Eq. (9) is valid for two-phase flows with $\phi > 0.8$ and $C_d$ is the drag coefficient and its magnitude depends on the particle Reynolds number:

$$C_d = \left\{ \begin{array}{ll}
\frac{2}{Re} (1 + 0.15 Re^{0.87}) & \text{Re}_p < 1000 \\
0.44 & \text{Re}_p \geqslant 1000
\end{array} \right.$$

where $\text{Re}_p = \frac{\phi \rho_l |\vec{V} - \vec{V}_p| d_p}{\mu_l}$ \hspace{1cm} (10)

The virtual mass force is proportional to the relative acceleration of the two phases and is written as (Drew and Lahey, 1993)

$$F_m = 0.5 \phi \rho_l \rho \frac{D}{\partial t} (\vec{V} - \vec{V}_p)$$

In Eq. (12) $D$ is the material derivative, such that, for the steady state case, the convective terms of the material derivative are retained.

The particle–particle interaction force is calculated as (Bouillard et al., 1989)

$$F_{\text{col}} = G(\phi_1) |\vec{V}_1 - \vec{V}_p| \phi_1$$

Here, $G$ is the particle–particle interaction modulus and it is calculated as

$$G = 1.0 \exp(-600(\phi_1 - 0.376))$$ \hspace{1cm} (14)

Eqs. (8)-(14) are not correlations developed for nano-sized particles. But, since there are no such correlations for nano-sized particles, it is assumed that it is reasonable to use them for nano-particles. Also, the importance of drag, virtual mass and particle–particle interaction forces in nanofluid will be discussed in Section 6.1.

2.3. Energy equations

Considering the base fluid and the particle phase as incompressible fluids, and neglecting the viscous dissipation and radiation, the energy equation is written as

$$\frac{\partial}{\partial x} \left( \rho f \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( \rho f \frac{\partial T}{\partial y} \right) = \frac{\partial}{\partial x} \left( \rho C_{\text{eff}} \frac{\partial T}{\partial x} \right) - h_v (T_m - T_p)$$ \hspace{1cm} (15)

$$\frac{\partial}{\partial x} \left( \rho p \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( \rho p \frac{\partial T}{\partial y} \right) = \frac{\partial}{\partial x} \left( \rho p \frac{\partial T}{\partial x} \right) - h_v (T_m - T_p)$$ \hspace{1cm} (16)

where $c_p, k_{\text{eff}}$ and $h_v$ are the heat capacity at constant pressure, temperature, effective thermal conductivity and volumetric interphase heat transfer coefficient, respectively. For mono-dispersed spherical particles $h_v$ can be calculated from

$$h_v = \frac{6(1 - \phi_1)}{d_p} h_p$$ \hspace{1cm} (17)

where $h_p$ is the fluid–particle heat transfer coefficient that should be calculated from empirical correlations. In the present study the fluid–particle heat transfer coefficient is calculated based on the Wakao and Kaguei (1982):

$$N_{up} = \frac{h_p d_p}{\kappa_1} - 2 + 1.1 \frac{\rho_p}{\rho_l} \frac{C_p}{C_1} \frac{d_p}{\mu_l}$$ \hspace{1cm} (18)

Here $Pr$ is the base liquid Prandtl number.

The effective thermal conductivities for liquid and particle phases are estimated as (Kuipers et al., 1992)

$$k_{\text{eff}} = \frac{k_{\text{liq}}}{\phi_l}$$ \hspace{1cm} (19)

$$k_{\text{eff}} = \sqrt{(1 - \phi_1) / \phi_l}$$ \hspace{1cm} (20)

where

$$k_{\text{liq}} = \left( 1 - \sqrt{(1 - \phi_1)} \right) k_l$$ \hspace{1cm} (21)

$$k_{\text{liq}} = \left( 1 - \phi_1 \right) (\omega A + |1 - \omega| B) k_l$$ \hspace{1cm} (22)

and

$$\Gamma = \frac{2}{1 - \frac{3}{2} B} \left( \frac{A(B - 1) - \frac{1}{B} - \frac{1}{2}}{1 - \frac{1}{B} - \frac{1}{2}} \right)$$ \hspace{1cm} (23)

with

$$B = 1.25 \left( \frac{1 - \phi_1}{\phi_l} \right)^{1/6}$$ \hspace{1cm} (24)

For spherical particles one can use

$$A = \frac{k_p}{k_l}$$ \hspace{1cm} and \hspace{1cm} $$\omega = 7.26 \times 10^{-3}$$ \hspace{1cm} (25)

Eqs. (18)-(25) are not correlations developed for nano-sized particles. But, due to the lack of such correlations for nano-sized particles, they are used in the present study.

2.4. Nußelt number definition

The Nußelt number is defined based on the temperature difference between the microchannel wall and the nanofluid mean (bulk) temperature:

$$Nu = \frac{(\partial T_m - T_w)}{T_w - T_m}$$ \hspace{1cm} (26)

where $T_m$, $T_w$, $q''$ are the convective heat transfer coefficient, microchannel hydraulic diameter and the wall convective heat transfer flux, respectively. Also, subscripts w and m stand for wall and mean, respectively.

The mean temperature for a two-phase fluid can be calculated from (Boulet and Moissette, 2002)

$$T_m = \frac{\sum_{n=1}^{N} \left( \int \rho u_c c_p T dA \right)}{\sum_{n=1}^{N} \left( \int \rho u_c c_p dA \right)}$$

(27)

where the integration is performed on the channel cross section. For two-phase flow according to Eqs. (15) and (16), wall convective heat transfer flux can be calculated as

$$q'' = \rho k_{\text{eff}} \frac{\partial T}{\partial y} |_{w} + \rho p k_{\text{eff}} \frac{\partial T}{\partial y} |_{w}$$ \hspace{1cm} (28)
According to the local Nusselt number, the average Nusselt number is

\[ \overline{Nu} = \frac{1}{L} \int_0^L \overline{Nu} \, dx \]  

(29)

2.5. Non-dimensionalization

Before the solution, all the governing equations are converted to non-dimensional form, using the following non-dimensional parameters:

\[ X = \frac{x}{D_h}, \quad Y = \frac{y}{D_h}, \quad U_i = \frac{u_i}{u_{in}}, \quad V_i = \frac{v_i}{u_{in}}, \quad P = \frac{p - p_{in}}{\rho u_{in}^2}, \quad \theta_i = \frac{T_i - T_{in}}{T_w - T_{in}} \]  

(30)

where \( i = l, p \) stands for the liquid and the particle phases.

2.6. Boundary conditions

Both phases enter the channel at the inlet with the same uniform axial velocity that is specified according to the flow Reynolds number. At the channel outlet, outflow velocity boundary condition is assumed for both phases. For liquid molecules the mean free path is 0.1–1 nm, so that the channel hydraulic diameter should be smaller than 1 \( \mu m \) to be in the slip region (Morini, 2005). Therefore, both for the phases the no-slip boundary condition at the walls is appropriate for the present study.

For thermal boundary conditions, it is assumed that the nanofluid enters the channel with 293 K and the isothermal walls have a temperature of 303 K. For the channel outlet, the outflow boundary condition is considered for both phases.

3. Numerical method

The non-dimensional form of the mass, momentum and energy conservation equations for liquid and particle phases along with the interphase correlations and boundary conditions are discretized using the finite volume method on the upper half of the channel (Patankar, 1980; Versteeg and Malalasekera, 1995). A non-uniform grid is employed in the computational domain. The grids are finer close to the wall and also in the channel entrance region using the cosine weighting function for control volume length and height. The power-law scheme (Patankar, 1980; Versteeg and Malalasekera, 1995) is used for the convection–diffusion term discretization. The set of the discretized equations is solved iteratively using the line by line method (Patankar, 1980; Versteeg and Malalasekera, 1995) and for solving the pressure–velocity coupling the well-known SIMPLE (Semi-Implicit Method for Pressure-Linked Equations) algorithm of Patankar (1980) is used. To use the SIMPLE algorithm, the pressure-correction equation is derived by combining the mass conservation equations for particle and liquid phases. Also, for accelerating the convergence of the SIMPLE algorithm, the under-relaxation for velocity and pressure is used. For convergence criteria, the sum of the scaled absolute residual for every parameter (mass, velocity and temperature) in all control volumes is calculated. The sum of the scaled absolute residuals for every parameter is restricted to be smaller than \( 10^{-6} \). For convergence criteria up to \( 10^{-6} \), the calculated average Nusselt number remains unchanged up to the third digit of the decimal. Thus, the convergence criterion is selected to be \( 10^{-6} \) in the present study.

4. Grid-independence study

To check for the independency of the results from the number of grid points used, a grid independence study is done by considering the amount of the calculated average Nusselt number. To do this, different numbers of grid points are used in the \( x \)- and \( y \)-directions. The results are shown in Table 1, where the flow Reynolds number is 1500. According to this study, the number of the grid points in \( x \)- and \( y \)-directions are considered 500 and 30 respectively in the present study.

5. Code validation

Due to the lack of experimental data for nanofluid flow in a parallel plate microchannel, the calculated average Nusselt numbers for the special case of pure water flow (\( \phi_p = 0.0 \)) at different Reynolds numbers are compared to corresponding available data in the literature in order to check the accuracy of the written computer code. For a single-phase fluid flow in an isothermally-heated parallel plate channel, the average Nusselt number is calculated as (Ebadian and Dong, 1998)

\[ \overline{Nu} = 7.55 + \frac{0.024x^{-1.14}}{1 + 0.0358Pr^{0.7}x^{-0.64}} \]  

(31)

where \( x = xD_h^2Re^{-1}Pr^{-1} \) is the non-dimensional axial length.

Table 2 shows the calculated average Nusselt number in the numerical simulation, the corresponding results from Eq. (31) and the difference between the two results for a wide range of Reynolds numbers. The results show that the agreement between the present numerical solution results and the existing solution from Eq. (31) is very good especially for smaller Reynolds numbers. According to Table 2 for lower Reynolds numbers the deviation is less than 1% and it is less than 4% for \( Re = 2000 \). After checking for the accuracy of the computer code, in the following section heat transfer and pressure drop results for different Reynolds numbers, nanoparticle diameters and concentrations are presented.

6. Results and discussion

Since in the present study the particle phase is considered as a continuum, its viscosity \( \mu_p \) has to be obtained. In fact, due to the

<table>
<thead>
<tr>
<th>No. of grid points in x-direction</th>
<th>No. of grid points in y-direction</th>
<th>Average Nusselt number at the wall</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>10</td>
<td>12.252</td>
</tr>
<tr>
<td>300</td>
<td>20</td>
<td>12.155</td>
</tr>
<tr>
<td>600</td>
<td>40</td>
<td>12.122</td>
</tr>
<tr>
<td>250</td>
<td>15</td>
<td>12.182</td>
</tr>
<tr>
<td>500</td>
<td>30</td>
<td>12.130</td>
</tr>
<tr>
<td>1000</td>
<td>60</td>
<td>12.116</td>
</tr>
</tbody>
</table>

Table 2

Comparison between average Nusselt number results from numerical simulations and Ebadian and Dong (1998) results for pure water flow at different Reynolds numbers.

<table>
<thead>
<tr>
<th>Re</th>
<th>Present study</th>
<th>Ebadian and Dong (1998)</th>
<th>Deviation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>7.66</td>
<td>7.621</td>
<td>0.51</td>
</tr>
<tr>
<td>50</td>
<td>7.756</td>
<td>7.739</td>
<td>0.22</td>
</tr>
<tr>
<td>100</td>
<td>7.924</td>
<td>7.934</td>
<td>-0.13</td>
</tr>
<tr>
<td>500</td>
<td>9.299</td>
<td>9.288</td>
<td>0.12</td>
</tr>
<tr>
<td>1000</td>
<td>10.843</td>
<td>10.633</td>
<td>1.93</td>
</tr>
<tr>
<td>1500</td>
<td>12.13</td>
<td>11.775</td>
<td>3.01</td>
</tr>
<tr>
<td>2000</td>
<td>13.238</td>
<td>12.775</td>
<td>3.62</td>
</tr>
</tbody>
</table>
lack of experimental data, the solid viscosity for a liquid–solid two-phase mixture is not available. So, for the first degree of approximation the following method is adopted in the present study: The corresponding pressure drop and the average Nusselt number of a highly dilute nanofluid with volumetric concentration of 0.00001 (which is quite close to pure water), is compared with that of pure water, for Re = 1500. Using the trial and error method, the value of viscosity in the solid phase of the highly dilute nanofluid is changed up to a point where the pressure drop and the average Nusselt number of the highly dilute nanofluid and the pure water are matched. In doing so, the viscosity of the solid phase reaches a value of $1.38 \times 10^{-3}$ Pa s. Under such conditions, the difference between the pressure and the average Nusselt number of the highly dilute nanofluid and that of pure water are 0% and 1.1%, respectively. In order to investigate the effect of Reynolds number of the flow, the same method of comparison between the pressure drop and the average Nusselt number of the highly dilute nanofluid and the pure water is made, under the condition of very low Reynolds number (equal to 100). Under these circumstances, the difference between the pressure drop and the average Nusselt number correspond to 0.04% and 1.4%, respectively. It can be concluded that the viscosity of the solid phase is independent of the Reynolds number. Also, Table 3 shows the sensitivity study of average Nusselt number on the solid viscosity magnitude for $Re = 100$, $\varphi_p = 0.01$ and $d_p = 100$ nm. As can be seen from Table 3 changing the solid viscosity, the amount of the average Nusselt number changes slightly. i.e., when the solid viscosity changes three orders of magnitude (from 0.01 to 0.00001) the average Nusselt number remains unchanged up to the first digit of the decimal. So, it can be concluded that the results are not very sensitive to the solid viscosity and it is not critical to know the exact magnitude for particle viscosity. With these discussions, the particle viscosity is considered to be $1.38 \times 10^{-3}$ Pa s in the present study.

6.1. Importance of the terms in the momentum equation

There are three interphase forces in the momentum equation (i.e. drag, virtual mass and particle–particle interaction forces). Table 4 shows the average Nusselt number for $Re = 300$, $d_p = 100$ nm and different term conditions. According to Table 4 virtual mass and particle–particle interaction forces do not have any effect on the average Nusselt number. But, the drag force affects the average Nusselt number slightly. Also, the drag force effect on the average Nusselt number increases with an increase in the nanoparticle volume concentration. So, it is possible to neglect the virtual mass and particle–particle interaction forces for nanofluid in the mathematical modeling.

6.2. Comparison with homogeneous model results

In this section, the two-phase modeling results are compared to the homogeneous modeling results of Santra et al. (2009). They reported average Nusselt numbers in different conditions for 100 nm copper–water nanofluid flowing inside a parallel plate channel. They used the channel height as a characteristic length scale to define Reynolds number (i.e., $Re_H$) and Nusselt numbers. Also, they used the temperature difference between the wall and the inlet fluid to define the heat transfer coefficient (in despite of Eq. (26)). So, to be able to compare the present two-phase results with the homogeneous modeling results of Santra et al. (2009), all the present results in this section are based to their definitions. Fig. 2 depicts the two-phase modeling results in comparison with the homogeneous modeling results for $Re_H = 100$ and 100 nm particles. It can be seen from Fig. 2 that the two-phase modeling results show higher heat transfer enhancement in comparison to homogeneous modeling results. Also, the heat transfer enhancement increases non-linearly with increase in nanoparticle volume.

<table>
<thead>
<tr>
<th>Volume concentration (%)</th>
<th>Considering all terms</th>
<th>Neglecting the drag term</th>
<th>Neglecting the particle–particle interaction term</th>
<th>Neglecting the virtual mass term</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12.343</td>
<td>12.385</td>
<td>12.343</td>
<td>12.343</td>
</tr>
<tr>
<td>3</td>
<td>15.507</td>
<td>15.637</td>
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Fig. 2. Comparison of percentage enhancement in average Nusselt number with respect to pure water flow for homogeneous and two-phase models at $Re_H = 100$, $d_p = 100$ nm and different nanoparticle volume concentrations. Two-phase results show higher heat transfer enhancement in comparison to homogeneous results.
concentration for two-phase modeling results, while it is almost linear for homogeneous modeling results. For instance, for 0.05 nanoparticle volume concentration, two-phase modeling shows 32.6% heat transfer enhancement, while it is 19% for homogeneous modeling results. For quantitative comparisons between the two-phase and homogeneous modeling results for different Reynolds numbers the data are collected in Table 5. The same behavior as the results show that the nanoparticle volume concentration distribution is uniform in the entire flow field. Such a result for nanoparticle distribution also has been reported by Akbarinia and Laur (2009) for a curved tube. Thus, considering the nanofluid as a homogeneous solution seems to be reasonable. Fig. 3 shows the effect of nanoparticles on the velocity field for the $Re = 1500$ and $\phi_p = 0.01$ case. In Fig. 3 the bottom contour lines show the velocity field for pure water flow in the computational domain, while the middle and the top ones show the velocity field for base liquid and nanoparticle phases in the nanofluid. From Fig. 3 it can be seen that the effect of nanoparticle on the velocity profile is very small and it slightly increases the hydrodynamic entrance length. This behavior is expectable due to very small size and concentration of the nanoparticles. This is the reason for a small increase in pressure drop for a nanofluid in comparison to the pure water. According to the velocity contour lines for base liquid and nanoparticle phases in the nanofluid in Fig. 3, it is clear that the relative velocity between the phases is very small and negligible.

Fig. 4 shows the effect of nanoparticles on the temperature field for the $Re = 1500$ and $\phi_p = 0.01$ case. In this figure the temperature contour line for pure water (the bottom plot) is compared to temperature contour lines for base liquid (middle plot) and nanoparticle (top plot) phases in the nanofluid. According to Fig. 4 nanoparticles increase the thermal boundary layer development considerably. This can be interpreted as increase in the thermal conductivity of the fluid due to the presence of the nanoparticles and as a result an increase in the heat transfer rate between the nanofluid and the microchannel wall. Also, the very small

### Table 5

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<tr>
<th>$\phi_p$ (%)</th>
<th>$Re = 500$</th>
<th>$Re = 1000$</th>
<th>$Re = 1500$</th>
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**Fig. 3.** Velocity contour lines for pure water (bottom plot) and nanofluid (base liquid phase in middle plot and nanoparticle phase in top plot) at $Re = 1500$, $\phi_p = 0.01$. Nanoparticles affect the velocity profile slightly and the relative velocity between the nanoparticles and base liquid is very small and negligible.
difference between the temperature profiles for base fluid and the nanoparticle phases is clear in Fig. 4.

6.4. Effect of particle volume concentration on heat and fluid flow

Fig. 5 depicts the effect of varying the particle volume concentration on the pressure drop for 100 nm particles. As it can be seen, the pressure drop increases slightly with increase of the nanoparticle volume concentration for all Reynolds numbers. This corresponds to the results reported by previous studies (e.g. Jung et al., 2009; Wu et al., 2009). This behavior is expectable since the nanoparticles have a very small effect on the velocity field as presented in the previous section. For instance, for \( \text{Re} = 1000 \) and \( \phi_p = 0.01 \), the percentage increase in pressure drop in comparison to pure water is 1.99%. This is one of the advantages of using nanoparticles rather than millimeter or micrometer sized particles.

Fig. 6 shows the particle volume concentration and flow Reynolds number effect on the average Nusselt number for 100 nm particles. It can be seen that the average Nusselt number increases with an increase in the nanoparticle volume concentration as well as an increase in the flow Reynolds number. By comparing Figs. 5 and 6 it is obvious that the increase in the Nusselt number is much higher than the increase in the pressure drop. So, this shows that nanofluids can be used as efficient working fluids in cooling systems. According to Fig. 6, increasing the Reynolds number and the nanoparticle volume concentration increases the heat transfer amount due to higher convection effects and higher nanoparticle
participation in increasing the effective thermal conductivity of the nanofluids, respectively. This behavior is in agreement with the experimental study of Wu et al. (2009) and the numerical study of Li and Kleinstreuer (2008) for microchannels.

Fig. 7 shows the nanofluid average Nusselt number normalized by the corresponding Nusselt number for pure water flow. From the figure it can be seen that for every Reynolds number, an increase in nanoparticle volume concentration increases the average Nusselt number ratio. This behavior is expected since the higher nanoparticle concentration increases the thermal conductivity of the nanofluid and thus causes an increase in the heat transfer rate. Also, Fig. 7 shows that for the same nanoparticle volume concentration, with an increase in Reynolds number, the average Nusselt number ratio decreases slightly. In other words, the effect of a specified amount of nanoparticles (constant nanoparticle volume concentration) on enhancing the heat transfer ratio is larger for the lower Reynolds numbers. For instance, at \( \phi_p = 0.01 \), the average Nusselt number ratio is 1.45 and 1.34 for \( Re = 200 \) and \( Re = 1600 \), respectively.

6.5. Effect of the nanoparticle diameter on heat transfer

Fig. 8 depicts the nanoparticle size effect on the amount of the heat transfer enhancement in comparison to pure water flow, at different volume concentrations and for \( Re = 500 \). It can be seen that an increase of the volume concentration and a decrease of the nanoparticle diameter increases the heat transfer enhancement. According to Fig. 8, the increase in heat transfer enhancement with a decrease in the nanoparticle size is not very pronounced especially for lower volume concentrations, since the particle sizes considered here are at the same order of magnitude. For instance, for a nanofluid with 0.01 nanoparticle volume concentration, for 100 nm and 30 nm particle sizes, the enhancement in heat transfer is 40.43% and 42.47%, respectively. The increase in heat transfer rate with a decrease in particle size was also reported by other researchers (Heris et al., 2007; Mirmasoumi and Behzadmehr, 2008b; Anoop et al., 2009; Akbarinia and Laur, 2009). Anoop et al. (2009) reported an increase in heat transfer enhancement with a decrease in particle size for experimental study of the alumina–water nanofluid flow in the developing region of a tube with nanoparticle diameters 45 nm and 150 nm. But, He et al. (2007) experimentally studied the TiO2 nanoparticle size effect on the heat transfer rate inside a tube. They performed the experiments for 95, 145 and 210 nm particles with 0.006 particle volume concentrations. Their heat transfer rate results did not show sensitivity to the nanoparticle size. They suggested a particle migration effect causing a depletion layer that is created due to migration of the larger particles to the channel center to explain this behavior. However, the continuum method used in the present study is not able to resolve a very small depletion layer of the order of the particle diameter close to the wall and the results show a uniform nanoparticle volume concentration distribution in the computational field.

7. Conclusions

Pressure drop and heat transfer due to copper–water nanofluid flow inside an isothermally-heated parallel plate microchannel is studied numerically for a wide range of Reynolds numbers, nanoparticle volume concentrations and nanoparticle diameters. To do this, the nanofluid flow is modeled using the Eulerian two-fluid model. In this method, the difference between the velocity and temperature for liquid and nanoparticle phases are considered and the governing equations for both phases are solved numerically using the finite volume method. It is observed that the relative velocity and temperature for base liquid and nanoparticle phases are very small and negligible. Thus, the liquid and the nanoparticles have almost the same velocity and temperature. Also, the nanoparticle volume concentration distribution is uniform in the computational domain. Therefore we conclude that considering the nanofluid as a homogeneous solution is reasonable.

The heat transfer enhancement results for two-phase modeling show higher magnitudes in comparison to the homogeneous modeling results. Such an observation also reported by other researches (Behzadmehr et al., 2007; Bianco et al., 2009; Fard et al., 2010; Lotfi et al., 2010). Thus, under-estimation of the heat transfer enhancement by homogeneous modeling seems to be related to the insufficient accuracy of the nanofluid thermophysical property models that are used in the homogeneous modeling. Also, the pressure
drop for nanofluids is slightly higher than the pressure drop for the pure water flow, while the average Nusselt number increases with increase in the Reynolds number and particle volume concentration. For the same nanoparticle volume concentration, the average Nusselt number ratio is higher for lower Reynolds numbers. Keeping all the other parameters constant, heat transfer enhancement is higher for the nanofluids with smaller nanoparticle sizes. However, this effect is not very pronounced at low nanoparticle volume concentrations. Also, the most important advantage of this method in comparison to homogenous modeling is that there is no need for effective thermophysical models for the nanofluid.

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References


